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J. V. Candy

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Sequential Model-Based Detection in a Shallow Ocean Acoustic Environment

James V. Candy

Lawrence Livermore National Laboratory, P.O. 808, L-156, Livermore, CA 94551, email: candy1@llnl.gov

Summary

A model-based detection scheme is developed to passively monitor an ocean acoustic environment along with its associated variations. The technique employs an embedded model-based processor and a reference model in a sequential likelihood detection scheme. The monitor is therefore called a sequential reference detector. The underlying theory for the design is developed and discussed in detail.

1. Introduction

It is well known that the ocean is a hostile, ever changing, medium requiring processors capable of adjusting to these changes to operate effectively [1-4]. The ocean acoustic monitor is a processor that passively listens and learns whether or not a target exists in the surveillance volume that is being monitored. In order to develop the monitor, we must incorporate our knowledge about the current ocean environment and its changes as time evolves. One way to accomplish this is through models that represent the ocean acoustics coupled with other a priori information to provide initial parameters for the processor. The development of the monitor uses a reference model developed during the calibration or learning phase and then listens for changes from the reference to declare an anomaly (possibly a target). Once an anomaly or change from the normal as characterized by the reference model is detected, the processor can then proceed to classify the target.

Using a normal-mode propagation model to represent the shallow ocean acoustics coupled with other a-priori parametric information, a sequential model-based detection approach is developed embedding the underlying physics into the monitor. Developing the monitor leads directly to a model-based, sequential likelihood ratio detection scheme. The approach we take is to develop a sequential reference detector based on the concept of establishing a model capable of representing the normal-mode environment with no target present. Using an embedded model-based processor (MBP), we adjust the embedded processor model based on noisy measurements made from a vertical array and essentially compare it to the calibration or reference model to achieve the required detection (see Fig. 1). In this paper we develop the underlying theory of model-based sequential detection applied to the ocean acoustic problem using a reference model.

Previous work in this area applied to ocean acoustic processing was based on developing a sequential innovations detector that detects changes from normal as well [5,6]. However, that monitor is based on testing the statistical properties of the innovations sequence to declare an anomaly, while the reference detector tests for changes from a reference model representing the normal or quiescent ocean. Each has its own advantages. In then next section, we discuss the underlying ocean acoustic models required to construct the processor while the required detection theory to construct the sequential reference monitor is developed in the subsequent section. There we show how the Wald sequential probability ratio test can be used to solve this problem [7].

2. Model-Based Ocean Acoustic Processing

In this section we discuss the basic models and concepts employed to develop the detection scheme as an alternative to the innovations-based approach
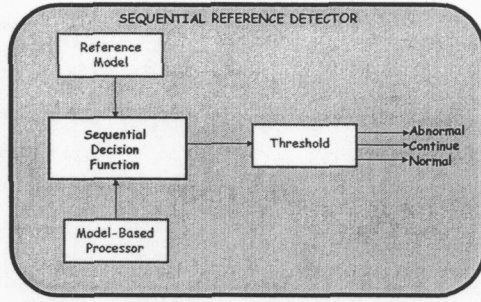
Figure 1: Sequential reference detector structure: reference model, MBP, decision function (Wald sequential probability ratio test) and threshold.

$$\mathbf{A}(z_\ell;\theta) \equiv \begin{bmatrix} \mathbf{A}_1(z_\ell;\theta) & & \mathbf{0} \\ & \ddots & \\ \mathbf{0} & & \mathbf{A}_M(z_\ell;\theta) \end{bmatrix}$$

with $\mathbf{A}_m(z_\ell;\theta) \equiv \begin{bmatrix} 1 & 0 \\ -\kappa_z(m) & 0 \end{bmatrix}$

and

$$\mathbf{c}'(z_\ell;\theta) \equiv \left[\beta_1(z_\ell)\ 0\ |\cdots| \ \beta_M(z_\ell)\ 0 \right]$$

with

$$\beta_m(z_\ell) = \frac{\varphi_m(z_s)}{\int \varphi_m(z_s)dz} \times \mathsf{H}_{\mathsf{o}}(\kappa_{\mathrm{r}}(m)z_s),\ m=1,\cdots,M$$

developed previously [5,6]. Since we are most interested in the shallow water problem, we select the basic normal-mode model and cast it into state-space form. We use the "depth-only," normal-mode *forward propagator* in Gauss-Markov form given by [4]

$$\varphi(z_{\ell+1}) = \mathbf{A}(z_\ell;\theta)\varphi(z_\ell) + \mathbf{w}(z_\ell) \qquad (1)$$

for z_ℓ the depth at the ℓth hydrophone with the corresponding pressure-field measurement model (scalar)

$$p(z_\ell) = \mathbf{c}'(z_\ell;\theta)\varphi(z_\ell) + v(z_\ell) \qquad (2)$$

where $\varphi, \mathbf{w} \in \mathsf{R}^{2M \times 1}$ and $\mathbf{c}' \in \mathsf{R}^{1 \times 2M}$ for M, the number of modes supporting the channel and $\mathbf{w}$, v are random, zero-mean, gaussian sequences with respective variances, $\mathbf{R}_{ww}(z_\ell), \sigma_{vv}^2(z_\ell)$. The corresponding process matrix, measurement and parameter vectors are $\mathbf{A} \in \mathsf{R}^{2M \times 2M}$, $\mathbf{c}' \in \mathsf{R}^{1 \times 2M}$ and $\theta \in \mathsf{R}^{N_\theta \times 1}$. The matrices and vectors are parameterized by

where H_{o} is the Hankel function. The embedded normal-mode model parameters are specified in terms of the *dispersion relation* evolving from the solution of the underlying wave equation as

$$\kappa \equiv \frac{\omega}{c(z)} = \kappa_z^2(m) + \kappa_r^2(m) \quad \text{for} \quad m=1,\cdots,M \quad (3)$$

with $\kappa, \kappa_z, \kappa_r$, the total, vertical and horizontal wave numbers, respectively, and $\omega, c(z)$, the corresponding temporal frequency and sound speed profile at depth z. We also define the model set corresponding to the forward propagator as the respective set of parameters for characterization as,

$$\Xi \equiv \left\{ \mathbf{A}(z_\ell;\theta), \mathbf{c}'(z_\ell;\theta), \mathbf{R}_{ww}, \sigma_{vv}^2, \theta \right\}$$

with

$$\theta \equiv \left[\beta, \kappa_z, \kappa_r, \omega, c(z), r_s, z_s \right]$$

the ocean acoustic parameters obtained through a combination of ocean experimental data and propagation model simulations [1].

Let us assume that through previous historical information about the shallow water region under investigation, we have developed a reasonable model under normal conditions with no targets present. For instance, after running ocean experiments and obtaining acoustic data in terms of sound speed profile, bottom parameters, etc., a set of simulation model parameters are obtained. With this experimental information available, an ocean acoustic propagation model can be constructed to analyze the propagation characteristics of the surveillance volume utilizing any of the existing

suite of algorithms available [1]. Using the calculated parameters of the propagation model, we implement the forward propagator based on the available information. We define the *reference model* given by

$$\bar{\varphi}(z_{\ell+1}) = \mathbf{A}(z_\ell;\theta)\bar{\varphi}(z_\ell)$$

$$p(z_\ell) = \mathbf{c}'(z_\ell;\theta)\bar{\varphi}(z_\ell) + v(z_\ell) \tag{4}$$

where $\bar{\varphi}(z_\ell)$ is defined as the modal reference trajectory which is created by running the forward propagator with no process noise (above). Since the reference model is developed during calibration from ambient ocean conditions, we assume that this model represents the "normal" ocean conditions and use it in our detection scheme to follow. For instance, this reference model can be the average or mean trajectory obtained by taking expected values of Eq. 4.

Suppose we wish to monitor the volume and construct a model-based innovations detector as discussed in [5,6]. The idea in obtaining a detection of a "change from normal" is to tune the processor for quiescent conditions with no target present. When a target enters the volume the embedded processor will no longer track the quiescent conditions, therefore it will detect the change indicating an "abnormal" condition or potential target. The innovations detector is an effective tool, but it gives us no information about the target acoustics. Although providing an alternate means for change detection, the reference detector can lead to a set of classification schemes by performing a multiple hypothesis tests based on "target" reference models instead of the quiescent or ambient models. So, with this in mind, the abnormal model is provided by the usual Gauss-Markov model of Eqs. 1 and 2; however, implemented within a model-based processor (Kalman filter) giving the filtered output as

$$\hat{\varphi}(z_\ell \mid z_\ell) = \hat{\varphi}(z_\ell \mid z_{\ell-1}) + \mathbf{k}(z_\ell)\varepsilon(z_\ell) \quad \text{[Estimator]}$$

$$\varepsilon(z_\ell) = p(z_\ell) - \mathbf{c}'(z_\ell;\theta)\hat{\varphi}(z_\ell \mid z_{\ell-1}) \quad \text{[Innovation]}$$

$$\mathbf{k}(z_\ell) = \tilde{\mathbf{P}}(z_\ell \mid z_{\ell-1})\mathbf{c}(z_\ell;\theta) / \sigma_{\varepsilon\varepsilon}^2(z_\ell) \quad \text{[Gain]}$$

$$\tag{5}$$

where $\hat{\varphi}(z_\ell \mid z_{\ell-1})$ is the modal function estimate at depth z_ℓ based on data up to depth $z_{\ell-1}$, $\varepsilon(z_\ell)$ is the innovations sequence, $\mathbf{k}$ is the gain function (Kalman gain) specified by predicted modal error covariance, $\tilde{\mathbf{P}}(z_\ell \mid z_{\ell-1}) = \text{Cov}(\varphi(z_\ell) - \hat{\varphi}(z_\ell \mid z_{\ell-1}))$. With these models and estimators in mind, we next develop the reference detection scheme.

2. Model-Based Reference Detection

In this section we develop the sequential reference detection scheme embedding the model-based processor of the previous section. Consider the following simple binary hypothesis test defined by:

$$H_0: \quad p(z_\ell) = \mathbf{c}'(z_\ell;\theta)\bar{\varphi}(z_\ell) + v(z_\ell) \quad \text{[Normal]}$$

$$H_1: \quad p(z_\ell) = \mathbf{c}'(z_\ell;\theta)\varphi(z_\ell) + v(z_\ell) \quad \text{[Abnormal]} \tag{6}$$

which forms the basis of the reference detector.

The development of the detector is based on the Neyman-Pearson criteria [7], which yields the optimal decision rule based on L measurements and results in the following likelihood ratio test with threshold T

$$L(\mathbf{P}_L) = \frac{\Pr(\mathbf{P}_L \mid H_1)}{\Pr(\mathbf{P}_L \mid H_0)} \underset{H_0}{\overset{H_1}{\underset{<}{>}}} T \tag{7}$$

where we define $\mathbf{P}_L$ as the set of depth measurements $\mathbf{P}_L \equiv \{p(z_1), p(z_2), \cdots, p(z_L)\}$. Applying Bayes' rule to this relation and replacing $L \to \ell$ gives

$$L(\mathbf{P}_\ell) = \frac{\Pr(p(z_\ell), \mathbf{P}_\ell \mid H_1)}{\Pr(p(z_\ell), \mathbf{P}_\ell \mid H_0)} = \tag{8}$$

$$\frac{\Pr(p(z_\ell) \mid \mathbf{P}_{\ell-1} \mid H_1)}{\Pr(p(z_\ell) \mid \mathbf{P}_{\ell-1} \mid H_0)} \times \frac{\Pr(\mathbf{P}_{\ell-1} \mid H_1)}{\Pr(\mathbf{P}_{\ell-1} \mid H_0)}$$

but recognizing the last ratio as $L(\mathrm{P}_{\ell-1})$ yields the sequential form as

$$L(\mathrm{P}_\ell) = L(\mathrm{P}_{\ell-1}) \times \frac{\Pr\left(p(z_\ell)|\mathrm{P}_{\ell-1}\,|\,\mathrm{H}_1\right)}{\Pr\left(p(z_\ell)|\mathrm{P}_{\ell-1}\,|\,\mathrm{H}_0\right)}. \qquad (9)$$

Now taking logarithms of Eq. 9, we obtain the *log-likelihood ratio* defined by $\Lambda(\ell) \equiv \ln L(\mathrm{P}_\ell)$

$$\Lambda(\ell) = \Lambda(\ell-1) + \ln \Pr(p(z_\ell)\,|\,\mathrm{P}_{\ell-1},\mathrm{H}_1) \\ -\ln \Pr(p(z_\ell)\,|\,\mathrm{P}_{\ell-1},\mathrm{H}_0) \qquad (10)$$

This result enables us to apply the Wald sequential probability ratio test [7] to give

$$\begin{aligned} \Lambda(\ell) &\geq \mathrm{T}_1 \\ \mathrm{T}_0 < \Lambda(\ell) &< \mathrm{T}_1 \qquad \text{[Continue]} \\ \Lambda(\ell) &\leq \mathrm{T}_0 \end{aligned} \qquad (11)$$

For our problem we must implement the monitor in terms of the normal-mode forward propagator. Since we have a Gauss-Markov model, then the required probability mass functions are gaussian, that is,

$$\Pr\left(p(z_\ell)\,|\,\mathrm{P}_{\ell-1},\mathrm{H}_1\right) = \\ \frac{1}{\sqrt{2\pi}\,\sigma_{\varepsilon\varepsilon}(z_\ell)} \exp\left\{-\frac{1}{2}\frac{\varepsilon^2(z_\ell)}{\sigma_{\varepsilon\varepsilon}^2(z_\ell)}\right\}$$

$$\Pr\left(p(z_\ell)\,|\,\mathrm{P}_{\ell-1},\mathrm{H}_0\right) = \\ \frac{1}{\sqrt{2\pi}\,\sigma_{vv}(z_\ell)} \exp\left\{-\frac{1}{2}\frac{\left(p(z_\ell)-\mathbf{c}'(z_\ell)\overline{\varphi}(z_\ell)\right)^2}{\sigma_{vv}^2(z_\ell)}\right\} \qquad (12)$$

Substituting the mass functions into $\Lambda(\ell)$ above, we obtain

$$\Lambda(\ell) = \Lambda(\ell-1) + \ln \frac{1}{\sqrt{2\pi}\,\sigma_{\varepsilon\varepsilon}(z_\ell)}$$

$$-\frac{1}{2}\frac{\varepsilon^2(z_\ell)}{\sigma_{\varepsilon\varepsilon}^2(z_\ell)} - \ln \frac{1}{\sqrt{2\pi}\,\sigma_{vv}(z_\ell)} \qquad (13)$$

$$+\frac{1}{2}\frac{\left(p(z_\ell)-\mathbf{c}'(z_\ell)\overline{\varphi}(z_\ell)\right)^2}{\sigma_{vv}^2(z_\ell)}$$

Including the known variables in the threshold we define the *sequential reference detector* as

$$\Lambda(\ell) = \Lambda(\ell-1) - \frac{1}{2}\frac{\varepsilon^2(z_\ell)}{\sigma_{\varepsilon\varepsilon}^2(z_\ell)}$$

$$+\frac{1}{2}\frac{\left(p(z_\ell)-\mathbf{c}'(z_\ell)\overline{\varphi}(z_\ell)\right)^2}{\sigma_{vv}^2(z_\ell)} \qquad (14)$$

and

$$\mathrm{T}_1(z_\ell) = \ln \mathrm{T}_1 + \ln \frac{1}{\sqrt{2\pi}\,\sigma_{\varepsilon\varepsilon}(z_\ell)} - \ln \frac{1}{\sqrt{2\pi}\,\sigma_{vv}(z_\ell)},$$

$$\mathrm{T}_1 = \frac{1-\mathrm{P_M}}{\mathrm{P_{FA}}}$$

$$\mathrm{T}_0(z_\ell) = \ln \mathrm{T}_0 + \ln \frac{1}{\sqrt{2\pi}\,\sigma_{\varepsilon\varepsilon}(z_\ell)} - \ln \frac{1}{\sqrt{2\pi}\,\sigma_{vv}(z_\ell)},$$

$$\mathrm{T}_0 = \frac{\mathrm{P_M}}{1-\mathrm{P_{FA}}} \qquad (15)$$

where $\mathrm{P_M}$, $\mathrm{P_{FA}}$ are the miss and false alarm probabilities. This completes the development of monitor.

3. Conclusions

In this paper we have developed a model-based, ocean acoustic monitor based on a reference detection scheme. It is shown that when a reference model is available, the monitor can be constructed and it essentially compares the output of a model-based processor with the reference model searching for an anomaly (possibly a target). The required theory is developed yielding the *sequential reference detector* design.

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